7. State Rolle's theorem and verify the Rolle's theorem for $f(x) = x^2 - 3x + 2$ in [0, 3].

Solution:

here,

$f(x) = x^2 - 3x + 2 \quad \forall \quad x \in [0, 3]$.

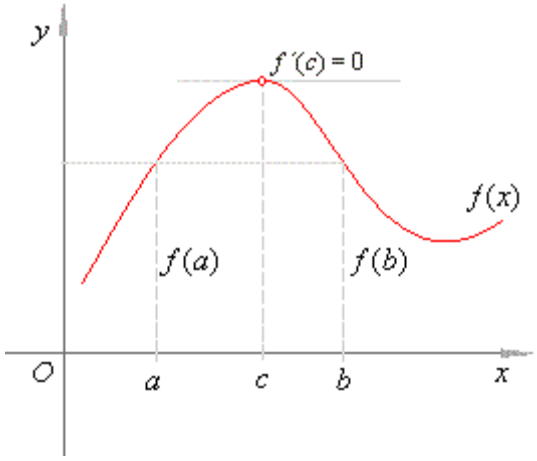
Rolle's theorem:

A function f(x) is:

i. continuous on [a, b]
ii. differentiable on (a, b)
iii. f(a) = f(b)

Then, there exists at least a point $c \in (a, b)$ such that,

$\therefore f''(c) = 0$.



Problem Part:

i. $f(x) = x^2 - 3x + 2$ is an polynomial function and polynomial functions are continous on their domain. So, f(x) is continous $\forall \ x \in [0, 3]$.

ii. $f'(x) = 2x - 3$ is defined $\forall \ x \in (0, 3)$.
So, f(x) is differentiable $\forall \ x \in (0, 3)$.

iii. Thus, f(x) satisfies both conditions of Rolle's Theorem so, by Rolle's Theorem there exists at least a point $c \in (0, 3)$ such that,

$f'(c) = 0$

or, $2c - 3 = 0$

or, $c = \frac{3}{2}$

$\therefore \quad c = \frac{3}{2} \in (0, 3)$

Hence, Rolle's Theorem is verified.